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GALAXY FORMATION AND COSMIC-RAY ACCELERATION IN A MAGNETIZED UNIVERSE

Loukas Vlahos,[1] Christos G. Tsagas,[2] and Demetrios Papadopoulos[1]
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ABSTRACT

We study the linear magnetohydrodynamic behavior of a Newtonian cosmology with a viscous magnetized fluid of finite conductivity and generalize the Jeans instability criterion. The presence of the field favors the anisotropic collapse of the fluid, which in turn leads to further magnetic amplification and to enhanced current-sheet formation in the plane normal to the ambient magnetic field. When the currents exceed a certain threshold, the resulting electrostatic turbulence can dramatically amplify the resistivity of the medium (anomalous resistivity). This could trigger strong electric fields and subsequently the acceleration of ultra–high-energy cosmic rays during the formation of protogalactic structures.

Subject headings: acceleration of particles — galaxies: formation — large-scale structure of universe — magnetic fields — turbulence

A wide variety of astrophysical and cosmological problems are currently interpreted on the basis of gravitational instability. The current large-scale structure and galactic evolution theories are some of the best known examples. Relatively few of the available studies, however, consider the role of magnetic fields, despite the widespread presence of the latter. Magnetic fields observed in galaxies and galaxy clusters are in energy equipartition with the gas and the cosmic rays. The origin of these fields, which can be astrophysical, cosmological, or both, remains an unresolved issue (Kronberg 1994; Han & Wielebinski 2002). If magnetism has a cosmological origin, as observations of microgauss fields in galaxy clusters and high-redshift protogalaxies seem to suggest, it could have affected the evolution of the universe (Grasso & Rubinstein 2001; Widrow 2002; Giovannini 2004). Studies of large-scale magnetic fields and their potential implications for the formation of the observed structure have been given by several authors (for a representative, though incomplete, list, see Thorn 1967; Jacobs 1968; Ruzmaikina & Ruzmakin 1970; Wasserman 1978; Papadopoulos & Esposito 1982; Zeldovich et al. 1983; Adams et al. 1996; Barrow et al. 1997; Tsagas & Barrow 1997; Jedamzik et al. 2000). Most of the early treatments were Newtonian, with the relativistic studies making a relatively recent appearance in the literature. A common factor among almost all the approaches is the use of the MHD approximation, namely, the assumption that the magnetic field is frozen into an effectively infinitely conductive cosmic medium. With few exceptions (Fennelly 1980; Jedamzik et al. 1998), the role of kinetic viscosity and the possibility of finite conductivity have been largely marginalized. Nevertheless, these aspects are essential for putting together a comprehensive picture of the magnetic behavior, particularly during the nonlinear regime. In this Letter, we consider a Newtonian expanding magnetized fluid and assume that both viscosity and resistivity are finite. At first, we look into the linear evolution of small inhomogeneities in the cosmic medium and examine how the field and the fluid viscosity affect the characteristic scales of the gravitational instability. We then discuss the electrodynamic properties of the collapsing fluid, the resulting magnetic amplification, and the formation of unstable current sheets. Central to our discussion is the concept of "anomalous resistivity," which is triggered by electrostatic instabilities in the plasma and can substantially reduce the electrical conductivity of the latter. We argue that such changes in the resistivity of the protogalactic medium will lead to the formation of strong electric fields during the galactic collapse. These fields can then accelerate the abundant free electrons and ions to ultrahigh energies.

Let us consider an expanding, incompressible, magnetized fluid with $p = p(\rho)$, where p and ρ are respectively the pressure and the density of the matter. This medium obeys the standard Newtonian MHD equations, which in comoving coordinates read

$$\frac{\partial \rho}{\partial t} = -3\frac{\dot{a}}{a}\rho - \frac{1}{a}\nabla\cdot(\rho\boldsymbol{u}), \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{\dot{a}}{a}\boldsymbol{u} - \frac{1}{a}(\boldsymbol{u}\cdot\nabla)\cdot\boldsymbol{u} - \frac{c_s^2}{a\rho}\nabla\rho + \frac{1}{a}\nabla\phi + \frac{1}{4\pi a\rho}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \frac{\nu}{a^2\rho}\nabla^2\boldsymbol{u}, \tag{2}$$

$$\nabla^2\phi = -4\pi G a^2\rho, \tag{3}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -2\frac{\dot{a}}{a}\boldsymbol{B} + \frac{1}{a}\nabla\times(\boldsymbol{u}\times\boldsymbol{B}) + \frac{\eta}{a^2}\nabla^2\boldsymbol{B}, \tag{4}$$

$$\nabla\cdot\boldsymbol{B} = 0. \tag{5}$$

In the foregoing, a is the cosmological scale factor, $\boldsymbol{u}$ is the fluid peculiar velocity (with $\nabla\cdot\boldsymbol{u} = 0$), $c_s^2 = dp/d\rho$ is the square of the sound speed, ϕ is the gravitational potential, $\boldsymbol{B}$ is the magnetic field vector, ν is the viscosity coefficient of the medium, and η is its electric resistivity. The system of equations (1)–(5) accepts a homogeneous solution with $\rho = \rho_0(t) \propto a^{-3}$, $\boldsymbol{B} = \boldsymbol{B}_0(t) \propto a^{-2}$, and $\boldsymbol{u} = \boldsymbol{u}_0 = 0$. This solution describes a weakly magnetized (i.e., $B_0^2/\rho_0 \ll 1$) Newtonian FRW universe, which also defines our unperturbed background.

Following equation (2), the magnetic effects are confined orthogonally to $\boldsymbol{B}$ (recall that $[(\nabla\times\boldsymbol{B})\times\boldsymbol{B}]\cdot\boldsymbol{B} = 0$), which ensures that there is no magnetic effect along the field's force lines. Given this, we align the background magnetic field along the z-axis of an orthonormal frame and consider its effects in the x-y plane. We do so by perturbing equations (1)–(5) around

[1] Department of Physics, Aristotle University of Thessaloniki, 541 24 Thessaloniki, Greece.
[2] Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, Wilberforce Road, University of Cambridge, Cambridge CB3 0WA, UK.

the zeroth-order FRW solution so that $\rho = \rho_0 + \rho_1$, $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1$, $\phi = \phi_0 + \phi_1$, and $\boldsymbol{u} \neq 0$. Assuming wavelike perturbations [i.e., $\rho_1(\boldsymbol{r}, t) = \tilde{\rho}_1(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$, $\boldsymbol{B}_1 = \tilde{\boldsymbol{B}}_1(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$, etc.] and using equations (1)–(5), the time derivative of equation (2) gives

$$\ddot{\boldsymbol{u}} = -\left(H + \frac{\nu k^2}{a^2\rho_0}\right)\dot{\boldsymbol{u}} + \left[8\pi G\rho_0 - \frac{k^2}{a^2}\left(c_s^2 + c_{\rm A}^2 + \frac{\nu H}{\rho_0}\right)\right]\boldsymbol{u} + i\left[\frac{c_s^2 H}{a\rho_0}\rho_1 - \frac{8\pi G a H}{k^2}\rho_1 + \frac{H}{2\pi a\rho_0}(\boldsymbol{B}_0\cdot\boldsymbol{B}_1) + \frac{k^2\eta}{4\pi a^3\rho_0}(\boldsymbol{B}_0\cdot\boldsymbol{B}_1)\right]\boldsymbol{k}, \quad (6)$$

where $H = \dot{a}/a$ is the Hubble parameter, $c_{\rm A}^2 = B_0^2/4\pi\rho_0$ is the Alfvén speed squared, and we have dropped the tildes for simplicity. Also, to reduce the algebra we have only considered perturbations orthogonal to the background magnetic field. The real component of the above provides a wave equation for the peculiar-velocity vector, which describes a damped oscillation. In particular,

$$\ddot{\boldsymbol{u}} = -\left(H + \frac{\nu k^2}{a^2\rho_0}\right)\dot{\boldsymbol{u}} + \left[8\pi G\rho_0 - \frac{k^2}{a^2}\left(c_s^2 + c_{\rm A}^2 + \frac{\nu H}{\rho_0}\right)\right]\boldsymbol{u}, \quad (7)$$

where the first term on the right-hand side shows the damping due to the expansion and of the fluid viscosity. The latter effect is scale dependent and vanishes on large enough scales (i.e., as $k \to 0$). The last term in equation (7) demonstrates the conflict between gravity on the one hand and fluid pressure and viscosity on the other. On large scales, gravity always wins and the perturbations collapse. Small-wavelength fluctuations, however, oscillate.

Accordingly, the magnetic presence adds to the supporting effects of pressure and viscosity only orthogonally to $\boldsymbol{B}_0$. This means that the first scales to collapse along the magnetic field lines are smaller than those normal to them. The two critical wavelengths are the associated Jeans scales

$$\lambda_\perp \simeq \sqrt{\frac{c_s^2 + c_{\rm A}^2 + \nu H/\rho_0}{8\pi G\rho_0}}, \quad \lambda_\parallel \simeq \sqrt{\frac{c_s^2 + \nu H/\rho_0}{8\pi G\rho_0}}, \quad (8)$$

orthogonal and parallel to $\boldsymbol{B}_0$, respectively. Overall, the magnetic presence induces a degree of anisotropy in the collapse. Note that for a pressureless, dustlike medium, the Jeans length along $\boldsymbol{B}_0$ depends entirely on the viscosity and the Hubble rate.

As the collapse proceeds, one expects the gradual formation of turbulent motions within the magnetized medium. The associated eddy viscosity is proportional to $\nu_{\rm turb} \sim \rho_0 u_1 l_{\rm mix}$, where u_1 is the velocity perturbation and $l_{\rm mix}$ is the turbulent mixing length (see, e.g., Biskamp 2003, p. 68). Assuming that u_1 reaches values close to $c_{\rm A}$ and that the mixing length is a fraction of the magnetically induced Jeans length (i.e., $l_{\rm mix} \ll \lambda_\perp$), and given the low thermal temperature of the postrecombination universe, the characteristic scaling on the velocities is $c_s^2 < \nu H/\rho_0 < c_{\rm A}^2$. Note that we have also adopted the typical values of $B \leq 10^{-7}$ G, $\rho_0 \geq 10^{-29}$ g cm^{-3}, and $H = 100\ h$ km s^{-1} Mpc^{-1}, where $0.4 \leq h \leq 1$. Then, $\lambda_\perp/\lambda_\parallel \sim [(c_{\rm A}^2\rho_0)/(\nu_{\rm turb}H)]^{1/2} \gg 1$, with $\lambda_\perp$ on the order of a (comoving) megaparsec. Following the standard structure formation scenarios, the initial collapse of this very large structure will be followed by successive fragmentation into smaller scale formations with characteristic lengths $\lambda \ll \lambda_\perp$. Moreover, as we will outline next, the anisotropy of the collapse will further increase the magnetic field trapped into the gravitating medium.

In the case of an almost spherically symmetric collapse, linear inhomogeneities in the magnetic energy density amplify in tune with those in the density of the matter, so that $\delta B^2 \propto \delta\rho$, where $\delta B^2 = B_1^2/B_0^2$ and $\delta\rho = \rho_1/\rho_0$ (Tsagas & Barrow 1998; Tsagas & Maartens 2000). Therefore, even within spherical symmetry, the formation of matter condensations in the post-recombination universe also signals the amplification of any magnetic field that happens to be present at the time. We have seen, however, that the generically anisotropic nature of the field will inevitably induce some degree of anisotropy in the collapse. Moreover, the magnetically induced anisotropy in the collapse will back-react and affect the evolution of the field itself. The magnetic evolution during the nonlinear regime of a generic, nonspherical protogalactic collapse has been considered by a number of authors (Zeldovich 1970; Zeldovich et al. 1983; Bruni et al. 2003; Siemieniec-Oziębło & Golda 2004; Dolag et al. 1999, 2002; Roettiger et al. 1999). The approaches are both analytical and numerical and agree that shearing effects increase the strength of the final field, while confining it to the protogalactic plane. Compared with the magnetic strengths of the spherical-collapse scenario, the anisotropic increase of B is stronger by at least 1 order of magnitude. Thus, protogalactic structures can be endowed with magnetic fields stronger than those previously anticipated.

So far, we have seen how the magnetic presence modifies the way gravitational collapse proceeds, by changing the overall stability of the magnetized fluid. This in turn affects the evolution of the field itself and can trigger a chain of nonlinear effects on certain scales. Next we will argue that this selective amplification of certain perturbative modes can play an important role during the nonlinear stages of protogalactic collapse, helping the instability to reach its saturation point. The current induced by the total field B is

$$\boldsymbol{J} = \frac{c}{4\pi}\boldsymbol{\nabla}\times\boldsymbol{B} = \frac{c}{4\pi}\alpha\boldsymbol{B}, \quad (9)$$

where $\boldsymbol{\nabla}\times\boldsymbol{B} = \alpha\boldsymbol{B}$ and α measures the magnetic torsion (see Parker 1993 for details). Initially α is small. However, the subsequent fragmentation of the protogalactic cloud will increase $\boldsymbol{\nabla}\times\boldsymbol{B}$ and strengthen the induced current. For example, a large-scale magnetic field with magnitude $B \sim 10^{-7}$ G at the time of the collapse will lead to $J \sim 10^2\alpha$. The latter can reach appreciable strengths for reasonable values of α.

When the current exceeds the critical value $J_c \sim \rho(e/m_p)c_s$, where $c_s \sim 10^4\sqrt{T({\rm K})}$ cm s^{-1}, m_p is the ion mass, and e is the electron charge, the excitation of low-frequency electrostatic turbulence will increase the resistivity of the medium by several orders of magnitude (Galeev & Sagdeev 1984; Kulsrud 1998). Note that the typical critical current is very small in the early postrecombination universe [i.e., $J_c \sim (\rho/m_p)ec_s \sim 10^{-10}$ statamp cm^{-2}]. Consequently, very small values of α will lead to $J > J_c$, thus making the plasma electrostatically unstable. The effect, which is known as "anomalous resistivity," can be explained through the development of current-driven electrostatic instabilities in the plasma. The latter lead to the excitation of waves and oscillations of different kinds. The absorption of these waves by the ions is an additional way of transferring momentum from the electrons to the ions, along with the usual momentum loss from the former species to the latter. The average momentum loss by an electron per unit time can be written as an effective collision term in the form $nm_e\nu_{\rm eff}\boldsymbol{u}_e = -\boldsymbol{F}_{\rm fr}$, where $\boldsymbol{F}_{\rm fr}$ is the average friction force and $n = \rho/m_p$ is the ambient number density of the plasma particles. The friction force is proportional to the linear growth rate of the electrostatic waves (γ_k) and the energy of the excited waves (W_k). The effective collision frequency is estimated to be $\nu_{\rm eff} = \omega_e(W_{\rm sat}/k_{\rm B}T)$. Then the anomalous resistivity will be

$$\eta_{\rm an} \sim \frac{\nu_{\rm eff}}{\omega_e^2} \sim \left(\frac{W_{\rm sat}}{k_{\rm B}T}\right)\frac{1}{\omega_e}\,, \tag{10}$$

where $\omega_e = 5.6 \times 10^4\sqrt{n}$ s^{-1} is the plasma frequency, $k_{\rm B}$ is the Boltzmann constant, and $(W_{\rm sat}/k_{\rm B}T) \sim 1$ is the saturated level of the electrostatic waves (Galeev & Sagdeev 1984). For certain types of current-driven waves, the anomalous resistivity is several orders of magnitude above the classical one, as confirmed in numerous laboratory experiments (Hamberger & Friedman 1968; Yamada et al. 1975).

This sudden switch to high electrical resistivity will inevitably lead to the formation of strong electric currents and therefore to a fast magnetic dissipation and intense plasma heating. The electric fields induced by the gravitational collapse will be $E \sim c_{\rm A}B/c + \eta_{\rm an}J_c \sim \eta_{\rm an}J_c$, given that $c_{\rm A}/c \ll 1$. Thus, in this scenario the gravitational collapse of the magnetized, postrecombination cloud amplifies the magnetic field and indirectly generates strong electric currents localized on the protogalactic plane. The anisotropy of the collapse enhances the local currents further and eventually drives the resistivity toward anomalously high values. The inevitable result is strong electric fields accelerating the abundant free electrons. The energy gain by an electron traveling a length $\lambda \sim \lambda_\perp$ is $W_{\rm kin} \sim eE\lambda_\perp \sim e\eta_{\rm an}J_c\lambda_\perp$, and the relativistic factor $\gamma = [1 - (v/c)^2]^{-1}$ is given by

$$\gamma - 1 = \frac{W_{\rm kin}}{m_ec^2} \sim \frac{e\eta_{\rm an}J_c\lambda_\perp}{m_ec^2} \sim \frac{e^2\eta_{\rm an}nc_s\lambda_\perp}{m_ec^2} \sim \frac{e^2B\sqrt{T}}{m_em_pc^2\sqrt{G}\sqrt{n}}$$
$$\sim 10^{11}\left(\frac{n}{10^{-4}\ {\rm cm}^{-3}}\right)^{-1/2}\left(\frac{T}{1\ {\rm K}}\right)^{1/2}\left(\frac{B}{10^{-7}\ {\rm G}}\right), \tag{11}$$

in cgs units. Recall that $J_c \sim enc_s$ and $c_s \sim 10^4\sqrt{T}$, and that $\lambda_\perp \sim B/nm_p\sqrt{G}$ when $c_s^2 < \nu H/\rho < c_{\rm A}^2$ (see eq. [8]). Also, we have set $W_{\rm sat}/k_{\rm B}T \sim 1$ in equation (10), which means that $\eta_{\rm an} \sim 1/(10^4\sqrt{n})$. Accordingly, the typical energy gain by a free electron can reach extremely high values within short timescales ($t_{\rm acc} \sim \lambda_\perp/c \sim 10^6$ yr), even for relatively weak magnetic fields. Clearly, one can extend this process to proton acceleration and show that protogalactic collapse can also produce ultra–high-energy cosmic rays.

We also anticipate a few particles drifting in and out these "primordial" current sheets (and the associated strong E-fields). If fragmentation has already taken place, these particles will diffuse along the different current sheets and possibly form the observed power-law distribution. The details of the acceleration processes, however, are beyond the scope of this Letter (see Arzner & Vlahos [2004] and Vlahos et al. [2004] for the diffusion of particles in many acceleration sites).

The role of unstable currents sheets along giant radio galaxies in the acceleration of cosmic rays has already been pointed out in the literature (Colgate et al. 2001; Nodes et al. 2003; Kronberg et al. 2004). Particles gain and lose energy (through synchrotron and inverse Compton emission) continuously, by traveling at speeds close to the speed of light. The suggestion made here is that ultra–high-energy cosmic ray acceleration and propagation may have started almost simultaneously with the formation of galaxies through the electrodynamic characteristics of the gravitational instability and continue, through the same processes, till today, since the previously described instability is active on all cosmic scales. It is also worth pointing out that the anomalous-resistivity mechanism can easily dissipate, in the form of bursty heating and particle acceleration, whenever $\eta_{\rm an}\nabla^2\boldsymbol{B} > \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B})$.

The role of cosmic magnetism during the early evolution of the first structures in our universe has been a subject of research and debate for many decades. Most of the available studies, however, operate within the limits of the MHD approximation; that is, they assume a highly conducting cosmic medium. As a result, the potential large-scale implications of a magnetic presence within a resistive environment are still relatively uncertain. In this Letter, we have considered a simple scenario that starts from the gravitational instability of a Newtonian, expanding, magnetized, viscous, and resistive fluid and discussed the implications of the field's presence during the early phases of what one might call the mild nonlinear protogalactic collapse. Focusing on the role of viscosity and especially on that of electrical resistivity, we have looked into the electrodynamic properties of the aforementioned gravitating medium and discussed issues such as current-sheet formation, anomalous resistivity, and particle acceleration on large scales.

We began by outlining the ways in which a magnetic presence and a finite fluid viscosity can alter the standard picture of gravitational instability. We then discussed how the preferential, anisotropic magnetic amplification will also increase the currents on the plane perpendicular to the main axis of the collapse. These gravitationally induced current sheets will in turn trigger electrostatic instabilities, which can then lead to anomalous resistivity values and subsequently to strong electric fields. We argue that the latter can be strong enough to accelerate the free electrons to ultrahigh energies.

The influence of magnetic fields on cosmic-ray propagation has been the subject of research in the past (see Sigl et al. 2004 and references therein). To the best of our knowledge, however, this is the first time that a direct connection between gravitational instability and cosmic-ray acceleration has been suggested and discussed. We have outlined the basic features of this connection in a simple scenario that involves only standard Newtonian magnetohydrodynamics. Given that we are in the postrecombination era and that the scales of interest are well within the horizon, we do not anticipate any general relativistic corrections. Special relativistic effects may need to be accounted for, but not before the particle velocities are an appreciable fraction of the light speed. Clearly, a detailed study of the acceleration mechanism proposed here should also consider nonlinear effects and the possible implications of a varying electrical resistivity. The latter has been treated as a slowly changing variable, relative to the acceleration timescale. In any case, the key requirement for this simple scenario to work is the presence of a magnetic field that is coherent on the scale of the collapsing protogalaxy. Our calculations argue that the required strength of this field is comparable to those observed in high-redshift protogalaxies. If such magnetic fields are widespread, as current observations indicate, their amplification during the nonlinear regime of galaxy formation can trigger a range of nontrivial effects. In this Letter we suggest that these effects can include the formation of strong large-scale current sheets and electric fields. The latter could act as driving sources for the cosmic rays observed in our universe today.

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